Planck's quantum hypothesis:

In 1900 scientist Max Planck assumed the following hypothesis

(i) The atom in the walls of the blackbody behave like 1-D. S.H.O each having a characteristic frequency ν and can have only those energy values given by

$E_n = nh\nu \rightarrow (1)$

$E_n = nE$, here $n = 1, 2, 3$ ....(integer).

(ii) In thermal equillibrium the emission and absorption of energy by the oscillator occur at equal rate.

The emission or absorption of energy take place in the form of descrete packets of energy known as quanta.

The energy (E) of a photon is proportional to its frequency ν.

i.e. $E = h\nu$, here $h = 6.626 \times 10^{-34} J - S$, the plank's constant.

Note: The 2nd hypothesis implies that, physical system can exist only in certain discrete energy states. i.e. energy is no longer contineous. This leads to the concept of atomicity of energy. Such discrete energy states are called quantum states, the integer n in equation (1) is called the quantum number.

Photon particle:

According to the plank's quantum hypthesis the energy of a photon is

$$E = h\nu = \hbar\omega \quad \left| \because \hbar = h/2\pi \;\&\; \omega = 2\pi\nu \right.$$

Since the photon is a relativistic particle and obeys the relativistic energy equation:

$$E^2 = p^2c^2 + m_0^2c^4$$

In fact the rest mass of the photon particle is zero i.e. $m_0 = 0$, then

$$E^2 = p^2c^2 \Rightarrow E = pc$$

So, the momentum $p = \frac{E}{c} = \frac{h\nu}{c} = \frac{h}{\lambda} \rightarrow (1)$

Since, from mass energy equivalence principle

We have, $E = mc^2$

$\therefore$ mass of the photon, $m = \frac{E}{c^2} = \frac{h\nu}{c^2} \rightarrow (2)$

Again, for wave motion $w = 2\pi\nu = 2\pi\frac{c}{\lambda} = ck \rightarrow (3)$

where $k = \frac{2\pi}{\lambda}$ is called wave vector.

Now, from eq. (1) $\Rightarrow$ momentum of photon

$$p = \frac{h}{\lambda} = \frac{h}{2\pi} \cdot \frac{2\pi}{\lambda} = \frac{h}{2\pi} . k = \hbar k \rightarrow (4)$$

So, the wave vector $\left|\vec{k}\right| = k = \frac{p}{\hbar} \rightarrow (5)$

Rest mass of the photon is zero

In relativistic case, the total energy of the photon particle is

$$E^2 = p^2c^2 + m_0^2c^4 \rightarrow (1) \qquad \left| k = \frac{2\pi}{\lambda} = \frac{2\pi p}{h} \right.$$

$\therefore$ Phase (or wave) velocity $\quad v_p = \frac{E}{p} \qquad \left| h\nu = \frac{2\pi\nu}{k} \text{P} \Rightarrow \frac{E}{p} = \frac{w}{k} \quad \therefore v_p = \frac{w}{k} \right.$

$$\therefore v_p = \frac{E}{p} = \frac{\left(p^2c^2 + m_0^2c^4\right)^{1/2}}{\left(p^2\right)^{1/2}}$$

$$= \left(\frac{p^2c^2 + m_0^2c^4}{p^2}\right)^{1/2}$$

$$= c\left[1 + \frac{m_0^2c^2}{p^2}\right]^{1/2} \rightarrow (2)$$

Now consider de-Broglie wave travelling with velocity c. This corresponds to the propagation of electro-magnetic waves. The velocity of the associated particle i.e. the photon is also 'c'.

Substitution $v_p = c$, in eq (2), we have

$$\frac{m_0^2c^2}{p^2} = 0$$

$$\Rightarrow \frac{m_0^2c^2\lambda^2}{h^2} = 0 \rightarrow (3) \quad \text{Since c, } \lambda \text{ \& h} \neq 0$$

So, eq. (3) $\Rightarrow m_0 = 0$

Thus the rest mass of the photon particle is zero.

Note: (1) $\Rightarrow E^2 = p^2c^2 \Rightarrow E = pc \Rightarrow h\nu = pc \Rightarrow p = \frac{h\nu}{c} = \frac{hw}{c}$ and

In a non-relativistic case, for a particle of mass *m*, moving with a velocity v, the total energy $\Rightarrow E = mc^2$ and momentum $p = mv$.

$\therefore$ Phase velocity of the associated de-Broglie wave

$$v_p = \frac{E}{p} = \frac{mc^2}{mv} = \frac{c^2}{v} \rightarrow (4)$$

As, $v < c$, the phase velocity $v_p > c$, velocity of light (photon) in vacuum. Eq. (4) is known as **de-Broglie** wave.

As a material particle can never have even the velocity of light, the phase velocity also exceeds the particle velocity.

Sumarising the important and u-
-seful relations of the photon particle, which has the frequency 'v' and wave length 'λ', we have already obtained.

Energy, $E = hv$

Rest mass, $m_0 = 0$

mass, $m = \frac{hv}{c^2}$

Momentum, $P = \frac{h}{\lambda} = k\hbar$